

The Standard Atmosphere

AE 1350



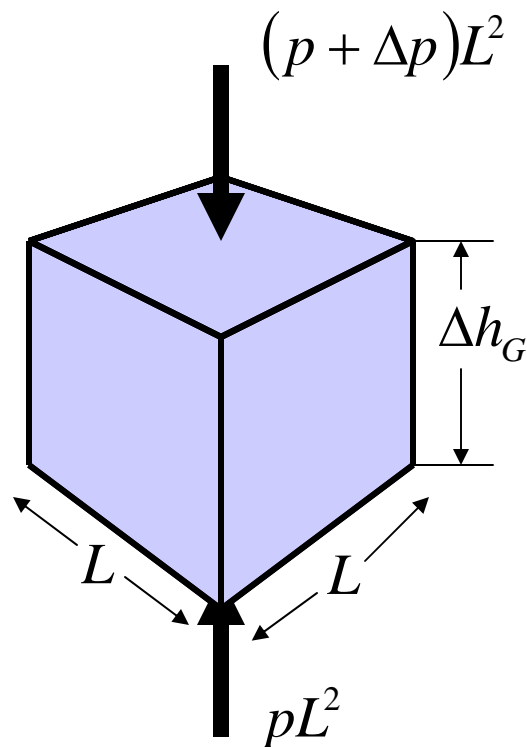
Weather and the Atmosphere



- Weather varies across location and time
- A standard atmosphere is defined for the Earth
 - We can say an airplane goes “100 mph” – and this will have *meaning*, since this implies we are talking about 100 mph in standard atmospheric conditions
 - Goes faster when air is less dense, has a tail wind, etc.
 - The standard atmosphere model is only a function of altitude (not latitude/longitude or time) and has no wind
- In terms of quantities we’ve been using
 - Pressure (p) in pounds per square foot, Newtons per m²
 - Density (ρ) in slugs per cubic foot, kilogram per m³
 - Temperature (T) degrees Rankine, Fahrenheit, etc.
 - Wind/gust velocity is zero

Hydrostatic Equation

- Standard atmosphere uses a model that accounts for pressure necessary to support the air above
- Consider a cube of air:

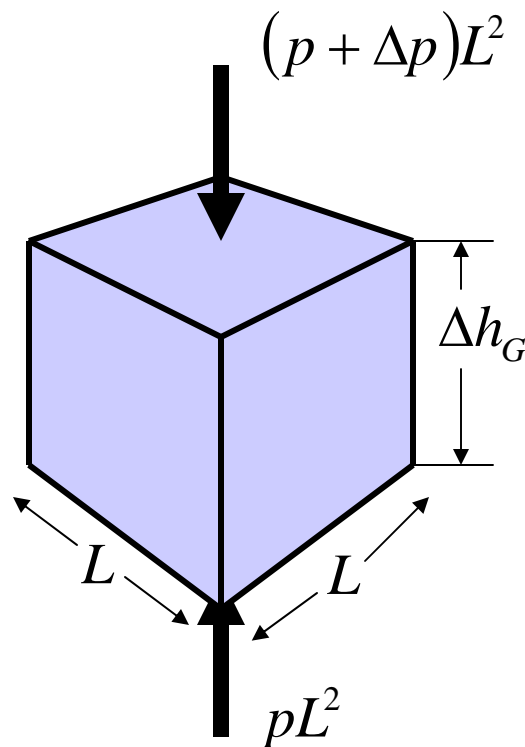


h is altitude

L is length/width of cube

Hydrostatic Equation

- Force must all balance
(from above and below plus gravity)



$$pL^2 = (p + \Delta p)L^2 + \rho g \Delta h_G L^2$$

g is acceleration due to gravity:

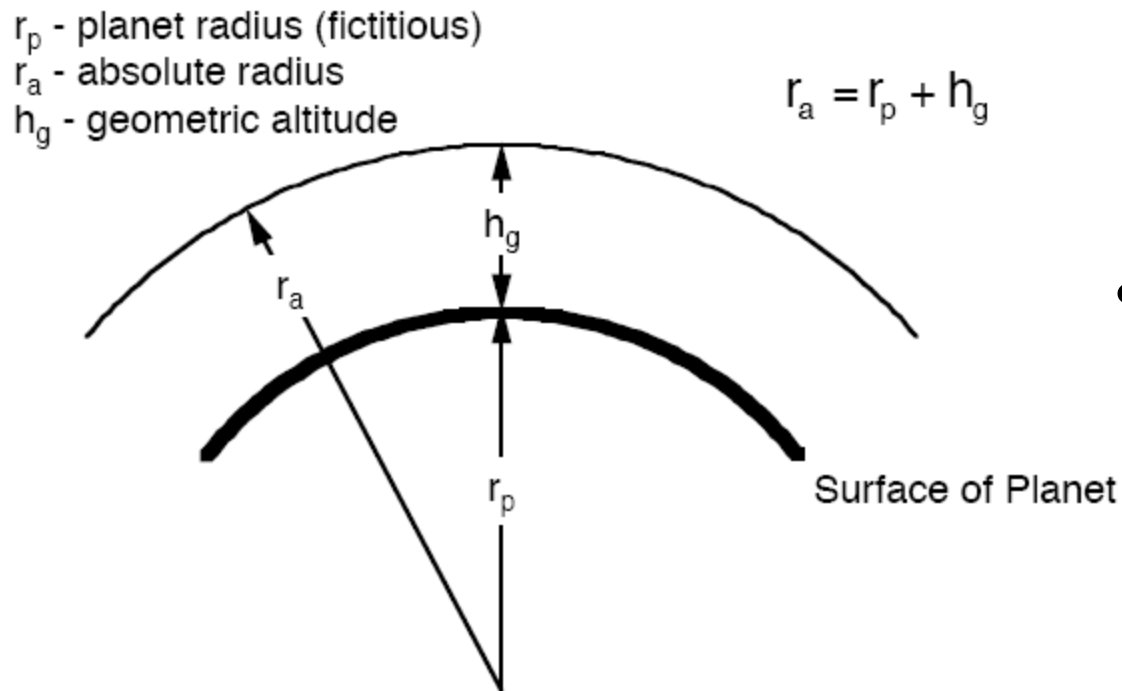
$$g = g_0 \left(\frac{r}{r + h_G} \right)^2$$

Gives us the hydrostatic equation:

go up and the pressure drops

$$\Delta p = -\rho g \Delta h_G$$

Geometric Altitude (h_g)



- Geometric altitude is the geometric height above a sea-level reference
- The gravitational attraction (which varies inversely with the square of absolute radius) at this reference is defined as g_0

Geopotential Altitude

- The inverse squared gravity law makes the differential equations a mess
- Fix by defining a new type of altitude: so gravity is effectively constant in the hydrostatic equation

$$\Delta p = -\rho g \Delta h_G \quad \text{becomes} \quad \Delta p = -\rho g_0 \Delta h$$

- To make this happen:

$$g \Delta h_G = g_0 \Delta h$$

$$\Delta h = \frac{g}{g_0} \Delta h_G$$

$$\Delta h = \frac{r^2}{(r + h_G)^2} \Delta h_G$$

So...

Both measures of altitude are nearly the same at low altitudes

Be careful which is being used!

Need to Relate Density and Pressure

- Model assumes air behaves like a perfect gas (intermolecular forces negligible)
- Equation of state:

$$p = \rho RT$$

where R is specific gas constant

$$R = 287 \frac{J}{kg \cdot ^\circ K}$$

$$R = 1716 \frac{ft \cdot lb}{slug \cdot ^\circ R}$$

Put it all together...

$$\Delta p = -\rho g_0 \Delta h$$

Hydrostatic equation

$$\frac{\Delta p}{p} = \frac{-\rho g_0 \Delta h}{p}$$

Divide by pressure

$$\frac{dp}{p} = \frac{-\rho g_0 dh}{\rho RT}$$

Use equation of state

$$\frac{dp}{p} = -\frac{g_0}{RT} dh$$

Cancel density

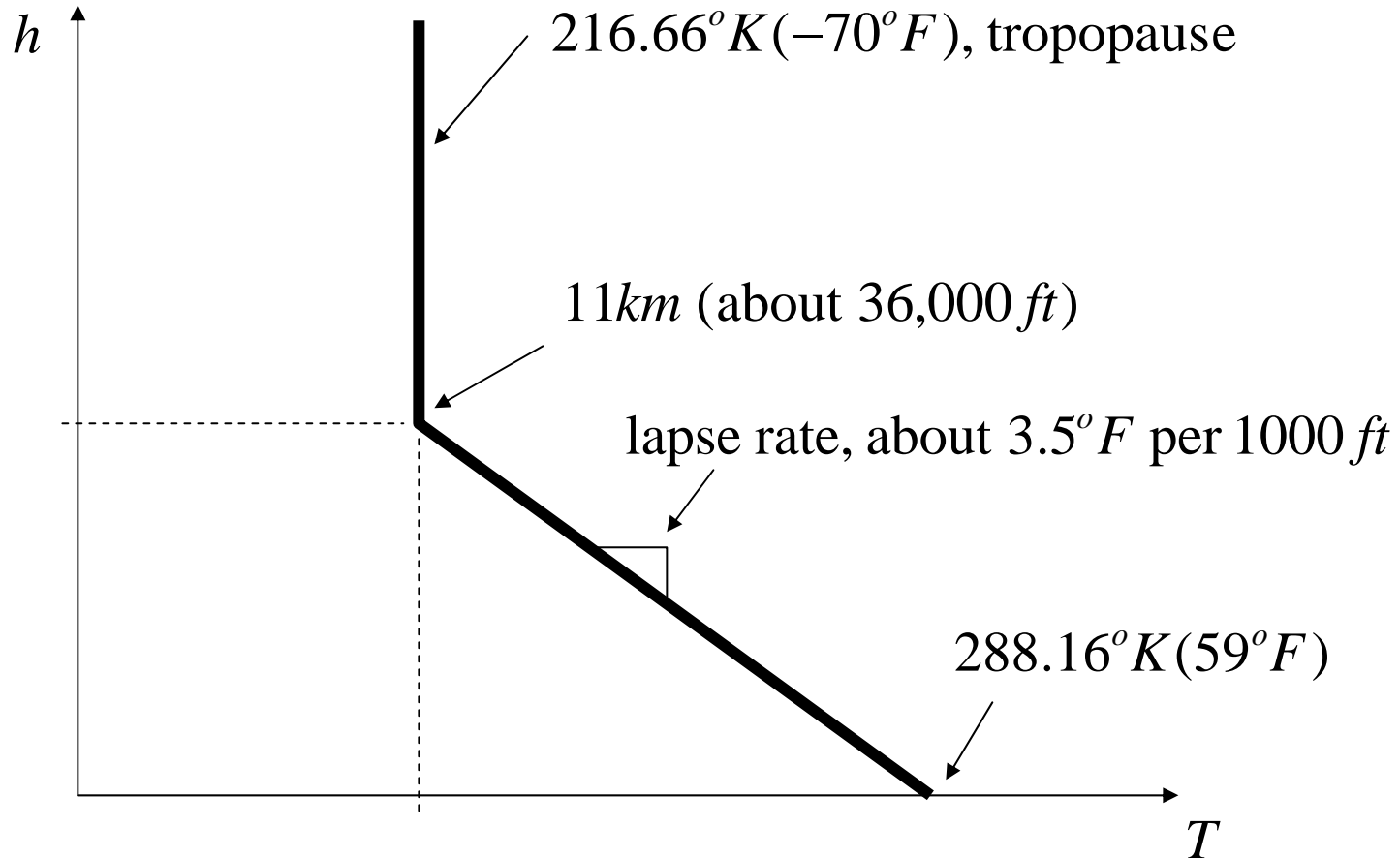
Now we have pressure as a function of temperature and altitude

The Standard Atmosphere Model

- The standard atmosphere *is* a temperature profile (function of altitude)
- Use $\frac{dp}{p} = -\frac{g_0}{RT}dh \longrightarrow$ pressure profile
- Use $\rho = \frac{p}{RT} \longrightarrow$ density profile
- Convert from geopotential altitude to regular altitude if needed

Temperature profile (first 20 km)

1976 U.S. Standard Atmosphere



In the Tropopause (above 36,000 feet)...

T is a constant

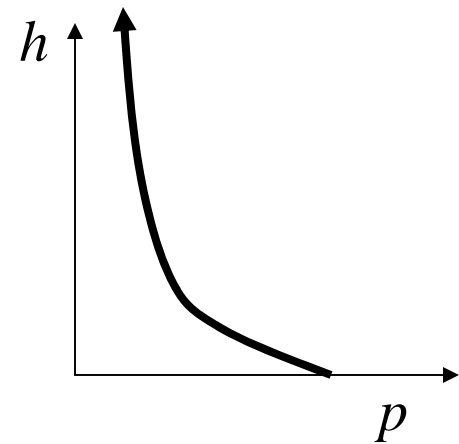
$$\frac{dp}{p} = -\frac{g_0}{RT} dh$$

$$\int_{p_1}^p \frac{dp}{p} = -\frac{g_0}{RT} \int_{h_1}^h dh$$

$$\ln \frac{p}{p_1} = -\frac{g_0}{RT} (h - h_1)$$

$$\frac{p}{p_1} = e^{-\frac{g_0}{RT}(h-h_1)}$$

$$\frac{\rho}{\rho_1} = e^{-\frac{g_0}{RT}(h-h_1)}$$



Below the Tropopause...

T is proportional to h , by lapse rate (a)

$$a = \frac{dT}{dh}$$

$$dh = \frac{dT}{a}$$

$$\frac{dp}{p} = -\frac{g_0}{RT} dh$$

$$\frac{dp}{p} = -\frac{g_0}{aR} \frac{dT}{T}$$

$$\int_{p_1}^p \frac{dp}{p} = -\frac{g_0}{aR} \int_{T_1}^T \frac{dT}{T}$$

$$\ln \frac{p}{p_1} = -\frac{g_0}{aR} \ln \frac{T}{T_1}$$

$$\frac{p}{p_1} = \left(\frac{T}{T_1} \right)^{-\frac{g_0}{aR}}$$

$$\frac{p}{p_1} = \frac{\rho RT}{\rho_1 RT_1} = \left(\frac{T}{T_1} \right)^{-\frac{g_0}{aR}}$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1} \right)^{-\left(\frac{g_0}{aR} + 1 \right)}$$

$$T = T_1 + a(h - h_1)$$

Other Constants Needed

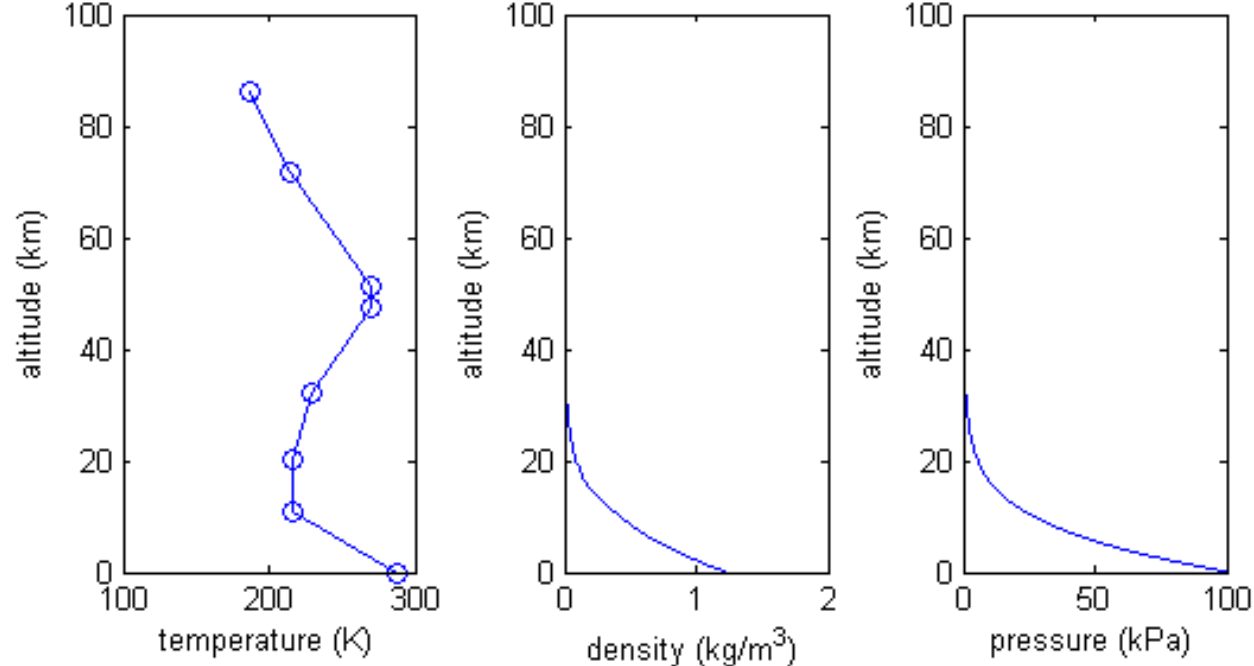
1976 U.S. Standard Atmosphere

- To get the pressure/density profiles, still need values at $h = 0$

$$p(0) = 101325 \frac{N}{m^2} = 2112.2 \frac{lb}{ft^2}$$

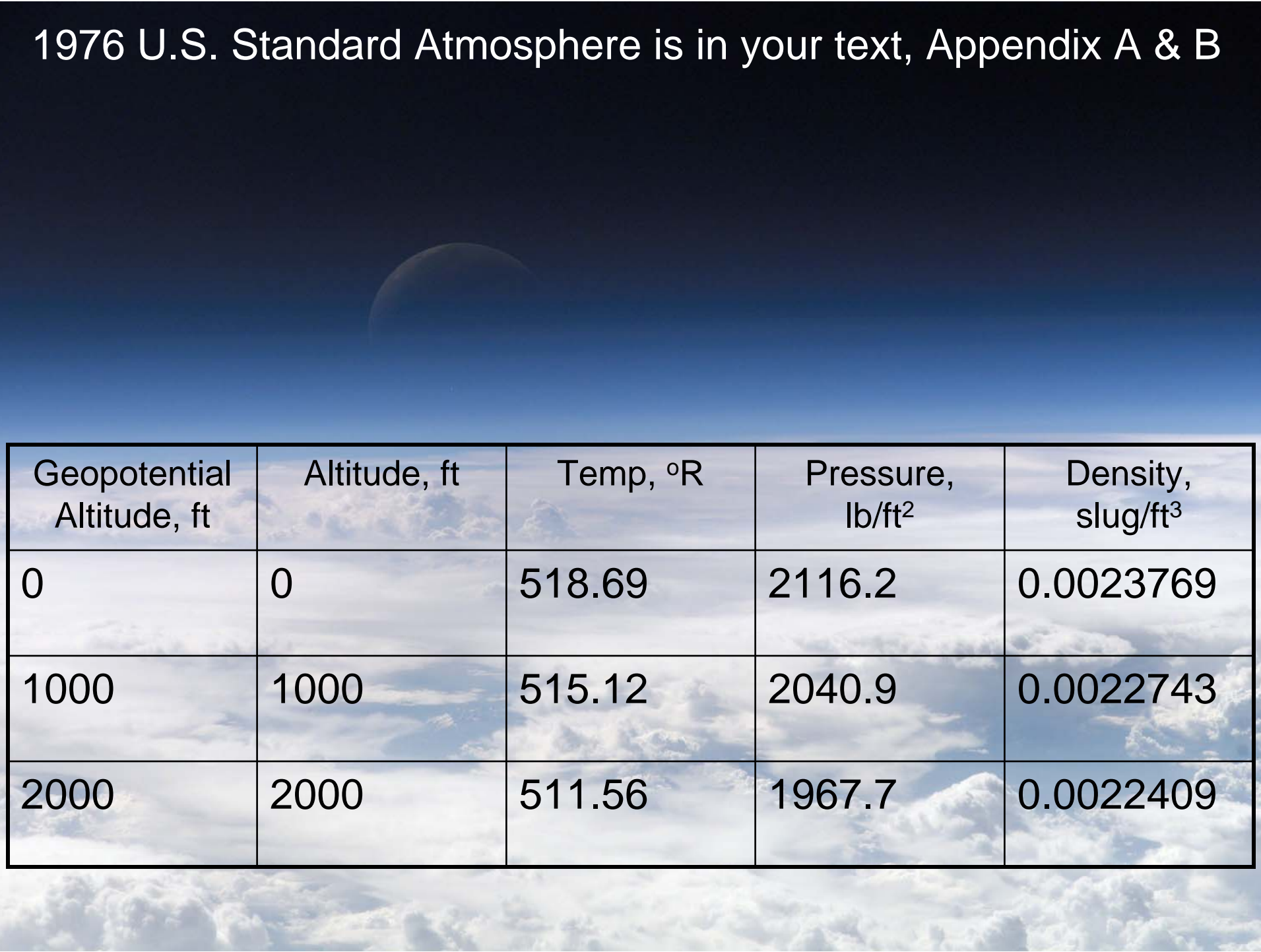
$$\rho(0) = 1.225 \frac{kg}{m^3} = 0.002377 \frac{slug}{ft^3}$$

The 1976 U.S. Standard Atmosphere



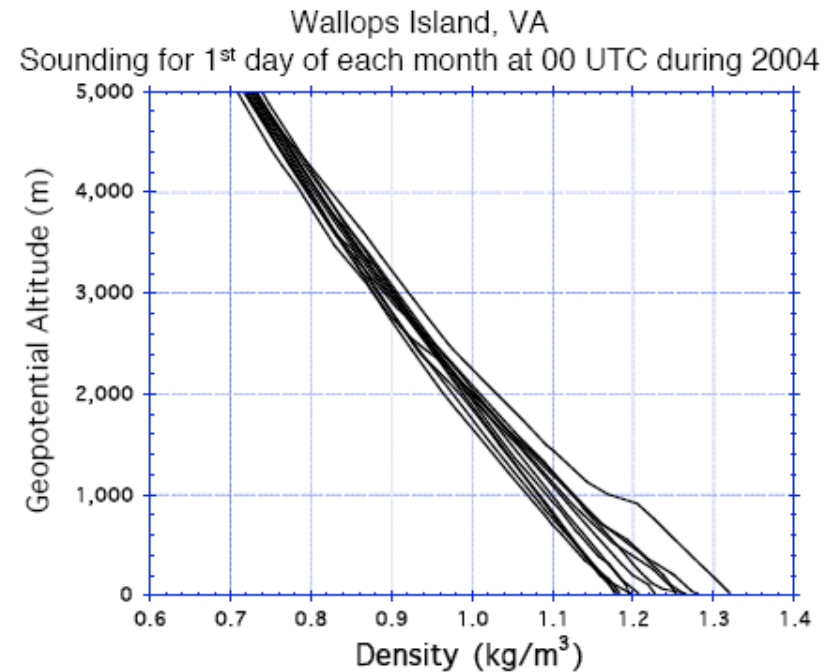
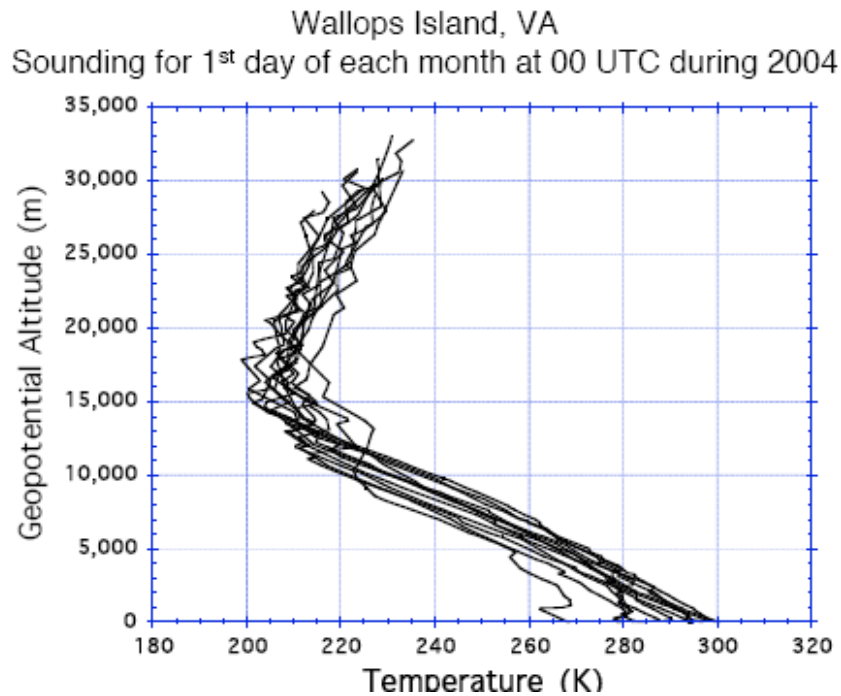
from Wikipedia

1976 U.S. Standard Atmosphere is in your text, Appendix A & B



Geopotential Altitude, ft	Altitude, ft	Temp, °R	Pressure, lb/ft ²	Density, slug/ft ³
0	0	518.69	2116.2	0.0023769
1000	1000	515.12	2040.9	0.0022743
2000	2000	511.56	1967.7	0.0022409

Flight Measured Atmospheric Properties



More Useful “Altitudes”

- There are useful metrics to compare current atmospheric conditions to the standard atmosphere

- Pressure altitude: Altitude in standard atmosphere with pressure corresponding to current pressure
- Density altitude: Altitude in standard atmosphere with density corresponding to current density
- Temperature altitude: ...you get it...